Int. J. Multiphase Flow, Vol. 1, pp. 89–101. Pergamon Press, 1973. Printed in Great Britain.

PARTICLE–GAS–SURFACE INTERACTIONS IN COLLECTION DEVICES

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(*Received* 10 *April* 1973)

Abstract—Removal of particles from a gas suspension via a surface, and subsequent regeneration of the surface, are analyzed. The deposition of the particles in the collection device is primarily due to electric field (in electrostatic precipitators) and to a combination of electric and centrifugal fields (in a cyclone separator). Proper design of particle separation devices, including surface regeneration, is related to these field forces and to other effects.

1. INTRODUCTION

An ideal system for the removal of particles from a gas is one in which the particles simply agglomerate until reaching a size such that free fall occurs. Specifically, particles large enough such that their terminal settling velocity in a given potential field exceeds the flow velocity of the gas, readily separate. The potential fields present may be gravitational, centrifugal, or electrical. If agglomeration is achieved simply, the cleaning system is small in dimension, low in power consumption, and high in particle collection efficiency (Soo 1973a).

Agglomeration in a particle cloud, with a distribution of particle sizes, can be achieved in an accelerating or decelerating fluid, which causes relative motion between large and small particles as a result of differences in their inertia (Soo 1967). Collision or scavenging of small particles by large particles leads to an increase in average particle size. An initial distribution in size is not necessary when sonic agglomeration techniques are employed. Sonic agglomeration is effective even for particles as small as 0.1 μm size. However, with all these techniques, high particle concentration (say, 2 g/m^3 for 1–10 μm particles) is needed for collisions to occur. As a result, large power consumption (1.5 kw/1000 m^3/sec of gas treated) is indicated by Green & Lane (1964).

One way to improve the efficiency is to increase the collision surface by introducing liquid droplets as done in a venturi scrubber. Such a scheme is feasible only if a liquid can be used. Another approach is to introduce dry surfaces for collection or agglomeration. A fluidized bed of pellets may be used but the pellet surface be cleaned, and surfaces that are easily cleaned must have simple geometry. One logical solution is direct use of the surface in a cyclone separator or an electrostatic precipitator for agglomeration. In these well known devices, the collecting surfaces may also be viewed as agglomerating surfaces which build up layers of particles over a length of time. The surfaces are then cleaned by removing the particles in lumps (or agglomerates). The power consumptions of these devices are in the range of a fraction of a kilowatt per 1000 m^3/sec of gas treated.

High collection efficiency by mutual agglomeration of particles is always limited by the lack of collisions among the particles as particle concentration decreases. Collisions with the walls of the system become more significant than mutual collision. Therefore, designing for the removal of particles from a gas requires knowledge of the interaction of the gas, the particles, and the wall surfaces.

We need to account for the settling out of particles by various field forces (gravitational, centrifugal and electrical), surface forces of adhesion; i.e. van der Waals force. Also, we must consider for particles the corresponding adhesion probabilities with a clean wall surface or a packed or moving bed of collected particles (Soo 1972a). These deposition mechanisms are opposed by lifting of particles in the shear flow field and lifting from a bed of particles. These effects were treated by Soo & Tung (1972) for the case of duct flow and by Soo & Rodgers (1971) for channel flow. We shall consider the general case and account for the splashing of particles produced by particles striking a layer of deposit (Bagnold 1951).

2. DIFFUSION UNDER FIELD FORCES

Rigorous calculation of a particulate system is currently limited to the cases of dilute suspensions and a packed or sliding bed. The reason is that in a dilute suspension, the scale of motion is large and the particle–particle interactions are weak; in a sliding bed the particle–particle interactions are strong but the displacements are small. In dilute suspensions (mean interparticle spacing greater than two mean diameters), the fluid motion is not strongly influenced by the presence of particles. The viscosity of the fluid phase in the mixture μ is the viscosity of the fluid intself $\bar{\mu}$. The viscosity of the particulate phase in the mixture arises from transport of momentum by diffusion, $\rho_p D_p$ (ρ_p is the particle phase density and D_p is the diffusivity of the particles). The momentum equation of the particulate phase of species (s) in a suspension with a distribution of particles sizes is given by:

$$\frac{d\mathbf{u}_p^{(s)}}{dt} = \frac{\partial \mathbf{u}_p^{(s)}}{\partial t} + \mathbf{u}_p^{(s)} \cdot \frac{\partial \mathbf{u}_p^{(s)}}{\partial \mathbf{r}} = F^{(s)}(\mathbf{u} - \mathbf{u}_p^{(s)}) + \mathbf{f}_p^{(s)\prime} + \mathbf{f}_p^{(s)} \qquad [1]$$

where t is time, $\mathbf{r}$ the spatial coordinates, $\mathbf{u}$ and $\mathbf{u}_p^{(s)}$ are the velocities of the fluid phase and species (s) of the particulate phase, respectively. $F^{(s)}$ is the inverse relaxation time for momentum transfer between the fluid and species (s) of the particles, ($F^{(s)} = 9\bar{\mu}/2a_s^2\bar{\rho}_p^{(s)}$ in the Stokes law regime), a_s is the radius of a particle of species (s), $\bar{\rho}_p^{(s)}$ is its density, $\bar{\rho}$ is the density of the gas such that $\bar{\rho}_p \gg \bar{\rho}$; $\mathbf{f}_p^{(s)\prime}$ is the force per unit mass due to the flow field and resistance to diffusion of momentum acting on species (s) and giving rise to the apparent viscosity of the particulate phase in the mixture and $\mathbf{f}_p^{(s)}$ is that due to field forces. For fully developed motion, or when inertial forces are smaller than the viscous and the field forces, the flux of species (s) is given by:

$$\rho_p^{(s)}(\mathbf{u}_p^{(s)} - \mathbf{u}) = \rho_p^{(s)}(\mathbf{f}_p^{(s)\prime} + \mathbf{f}_p^{(s)})/F^{(s)} \qquad [2]$$

where $\rho_p^{(s)}$ is the density of species (s) in the particle cloud, assuming that $\mathbf{u}$ is nearly the mean velocity of the dilute suspension.

Furthermore, since the suspension is dilute, the diffusion equation of species (s) is given by:

$$\frac{\mathrm{d}\rho_p^{(s)}}{\mathrm{d}t} = \frac{\partial \rho_p^{(s)}}{\partial t} + \mathbf{u}\cdot\frac{\partial \rho_p^{(s)}}{\partial \mathbf{r}} = -\nabla \cdot [-D_p^{(s)}\nabla\rho_p^{(s)} + \rho_p^{(s)}(\mathbf{u}_p^{(s)} - \mathbf{u})]. \quad [3]$$

For a thin particle bed, sliding over a wall, the mass balance includes the deposition rate resulting in the thickness of the bed. In the force balance, inertia can be neglected and only shear stress and field forces need be considered (Soo 1973b).

3. LIFT AND DEPOSITION AT THE BOUNDARY

Considerations of physical effects at the boundary of a flowing suspension must include diffusion of species (s) of particles, field forces per unit mass $\mathbf{f}_p^{(s)}$, adhesion probability $\sigma_w^{(s)}$ for deposition, surface force per unit mass $\mathbf{f}_w^{(s)}$, adhesion probability $\sigma_w^{(s)}$ prior to the completion of a monolayer deposition, and lift force $\mathbf{f}_L^{(s)}$ per unit mass due to fluid shear. In general, $\mathbf{f}_L^{(s)}$ should be included over the whole shear layer and it is insignificant for both $2a_s \ll \delta$, and $2a_s \gg \delta$, (δ is the boundary layer thickness of the fluid phase). The flux of particles produced by the erosion from a bed is given by:

$$\begin{pmatrix}\text{Probability}\\ \text{of lift}\end{pmatrix} \times \begin{pmatrix}\text{Mass per unit area of}\\ \text{particles layer}\end{pmatrix} \times \begin{pmatrix}\text{Frequency of lifting}\\ \text{of particles}\end{pmatrix}.$$

Since the frequency of lifting is given by the lift velocity $(f_L^{(s)}/F^{(s)})$ divided by the mean distance between successive particles lifted, the flux of particles produced by erosion is thus: $\sigma_w^{(s)'}\rho_{pb}^{(s)}f_L^{(s)}/F$ at the wall, where the probability $\sigma_w^{(s)'}$, accounts for the difference between bed density $\rho_{pb}^{(s)}$ and the above quantity (mass per unit area divided by the mean distance between lifted particles). With $f_L^{(s)}$ thus accounted for, and the boundary condition is (Soo & Tung 1972; Soo 1973b);

$$-D_p^{(s)}\frac{\partial \rho_p^{(s)}}{\partial n}\bigg|_w = (1-\sigma^{(s)})\left(\frac{f_p^{(s)}}{F^{(s)}}\right)\rho_{pw}^{(s)} - \sigma_w^{(s)}\left(\frac{f_w^{(s)}}{F^{(s)}}\right)\rho_{pw}^{(s)} - [(1-\sigma^{(s)})\rho_{pw}^{(s)} - \sigma_w^{(s)'}\rho_{pb}^{(s)}]\frac{f_L^{(s)}}{F} + J_{\text{splash}} \quad [4]$$

where n is the coordinate normal to the wall, $\rho_{pw}^{(s)}$ is the density of the suspension over the layer deposited on the wall, $f_p^{(s)}$ are directed toward the wall. The last term in [4] gives the effect of splashing by larger particles of species (r) colliding with the deposit layer, considered in the next section.

For cases which include an erodible bed of solid particles, the lift force per unit mass $f_L^{(s)}$ is given by (for coordinate y in the $\mathbf{n}$ direction):

$$f_L^{(s)} = N_s^{(s)}\frac{u_o}{L}\left|\frac{L}{u_r^{(s)}}\frac{\mathrm{d}u}{\mathrm{d}y}\right|^{1/2} u_r^{(s)} \quad [5]$$

where $u_r^{(s)}$ is the local relative velocity of particles to gas, u_o is the characteristic fluid velocity, and L is a characteristic length of the flow system. The relative velocity is given by:

$$u_r^{(s)} = u_p^{(s)} - u + \Delta u^{(s)} \qquad [6]$$

where $\Delta u^{(s)}$ is the relative velocity due to fluid shear alone ($\Delta u^{(s)} \simeq a_s\, du/dy$), and the shear response number, N_s, is given by data of Graf & Acaroglu (1968) as:

$$N_s^{(s)} = \tfrac{3}{4} c_1 \frac{\bar{\rho}}{\bar{\rho}_p^{(s)}} \left(\frac{\mu}{L u_o \bar{\rho}}\right) \left(\frac{L}{2a_s}\right)^{3/2} \qquad [7]$$

for substantial relative motion based on experimental results, $c_1 \approx 50$.

The force of adhesion of particles to a clean surface or to a surface with a layer of deposited particles, influences further deposition of particles. A survey by Soo (1973b) shows that the adhesive forces are either electrical or liquid in origin. The electrical forces include those due to contact potential difference, dipole effect, space charges, and electronic structure. Krupp (1967) showed that the adhesive force between a solid plane and spherical solid particle is proportional to the particle radius. Adhesion probabilities σ have been determined only in isolated cases, notably by Löffler & Muhr (1972). Current data show $\sigma \simeq 10^{-4}$ for adhesion by van der Waals force, and $0.5 \gtrsim \sigma \gtrsim 0.05$ for attachment by field forces.

The relative motion of particles at the boundary is given by

$$u_{pw}^{(s)} = L_p^{(s)} \left(\frac{\partial u_p^{(s)}}{\partial y}\right)\bigg|_w \qquad [8]$$

where $L_p^{(s)}$ is the interaction length of particles (s) with the fluid,

$$L_p^{(s)} = \langle (U_p^{(s)})^2 \rangle^{1/2} / F^{(s)}$$

where

$$\langle (\Delta u_p^{(s)})^2 \rangle^{1/2}$$

is the intensity of relative motion between the particle and the fluid (Soo 1967).

4. EFFECT OF SPLASHING

Bagnold (1951) observed the splash of particles when hitting a layer of non-sticking particles. Seman & Penny (1965) also noted that particles striking a layer of electrostatically deposited dust cause splashing and reentrainment. Such an effect is readily accounted for by considering the particle–particle interaction when a cloud of particles of species (r) collides with a deposit layer including species (s) and bed density $\rho_{pb}^{(s)}$. We note that:

(a) In a monodispersed suspension, because particles cannot be distinguished, any splashing constitutes exchange of particles with the deposited layer.

(b) Splashing is produced mainly by particles larger than those of the species (s) under consideration in the deposit layer (Soo 1967, 1973b) since the fraction of kinetic energy of a particle (r) transferred by collision with a particle (s) is proportional to the ratio of their masses, m_r/m_s.

For particles (r) travelling at velocity $u_{pw}^{(r)}$ colliding with a bed including species (s), the inverse relaxation time, $F^{(sr)}$, for momentum transfer was given by Soo (1967) as:

$$\rho_p^{(r)}F^{(rs)} = \rho_{pb}^{(s)}F^{(sr)}$$

and

$$F^{(sr)} = \tfrac{3}{4}\eta^{(sr)} \frac{[\sqrt{(a_s/a_r)} + \sqrt{(a_r/a_s)}]^2 u_{pw}^{(r)}\rho_p^{(r)}}{\sqrt{(a_s/a_r)}\sqrt{(\rho_p^{(s)}\rho_p^{(r)})}[\sqrt{(m_s/m_r)} + \sqrt{(m_r/m_s)}]}$$

where $u_{pw}^{(r)}$ is the velocity of the particle cloud (r) at the wall, $\eta^{(sr)}$ is the fraction impacted of a particle (s) by cloud (r) and $\eta^{(sr)} \simeq 1$, a_s and a_r are the radii of particles of the respective species, m_s and m_r their masses, and $\bar{\rho}_p$ is the material density of each species. The flux of (s) particles produced by splashing of particles of various species (r) at probability $\sigma_s^{(sr)}$ is given by:

$$J_{\text{splash}} = \sigma_s^{(sr)} \frac{\rho_{pb}^{(s)}}{F^{(s)}} \sum_{(r)} F^{(sr)} u_{pw}^{(r)} \propto \sigma_s^{(sr)} \frac{\rho_{pb}^{(s)}}{F^{(s)}} \sum_{(r)} \rho_{pw}^{(r)} u_{pw}^{(r)2}/a_r\bar{\rho}_p \tag{9}$$

for $m_r > m_s$. Equation [9] gives the last term of [4]. Note that the flux of species (s) enhanced by splashing is proportional to a force per unit mass represented by $u_{pw}^{(r)2}/a$; with u_{pw} given by [8]. Splashing is insignificant when L_p of a species is small. Note also that we have neglected the $(s–r)$ collision in the dilute suspension in [1] but accounted for $(s–r)$ collision in the layer of deposit.

5. APPLICATION TO ELECTROSTATIC PRECIPITATORS

The basic relations of density distribution and deposition at a surface are demonstrated in figure 1. The system consists of corona wires and collector plates as shown in figure 1(a); a simplified representation is given in figure 1(b). The suspension is flowing with velocity U in the x-direction, and a drift velocity KE in the y-direction. K is the mobility of the particles ($K = (q/m)/F$) for charge q and mass m of a particle, and E is a uniform electric field. Figure 1(b) also represents the situation in a collector passage such as in a two-stage precipitator as is shown in figure 1(c). The slug flow is a good approximation, especially for cases of large interaction length giving a large relative velocity from [8].

The diffusion equation of the particles of a given species takes the form:

$$Pe\frac{\partial \rho_p}{\partial x^*} + \alpha_e\frac{\partial \rho_p}{\partial y^*} = \frac{\partial^2 \rho_p}{\partial y^{*2}} \tag{10}$$

for $x^* = x/b$ and $y^* = y/b$. Pe and α_e are the Peclet number and a parameter correlating drift and diffusion, respectively, and

$$Pe = \frac{bU}{D_p} \qquad \alpha_e = \frac{KEb}{D_p} = \left(\frac{q}{m}\right)\frac{Fb}{FD_p}.$$

Using these definitions, the boundary conditions [4] are (Soo & Rodgers 1971; Soo 1972b):

$$@\ y^* = 0, \qquad \frac{\partial \rho_p}{\partial y^*} = \alpha_e\rho_p$$

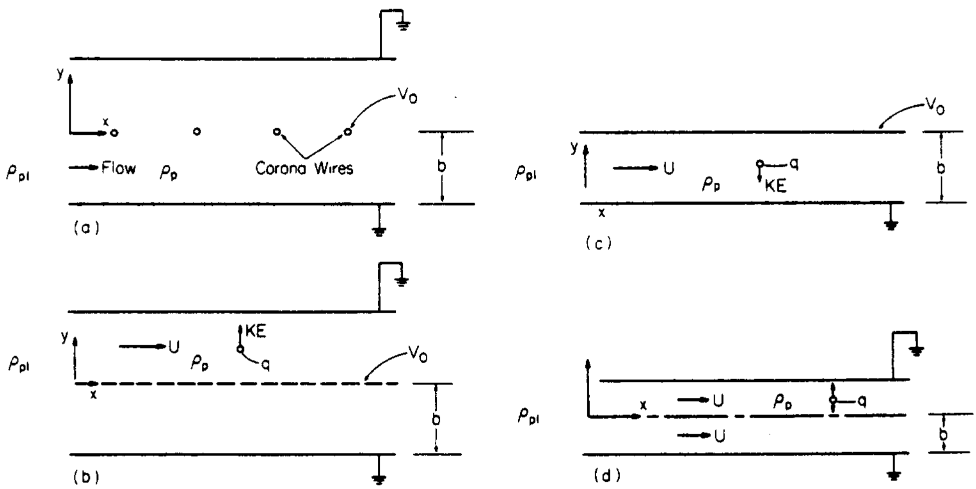

Figure 1. Field and boundary configurations of electro-precipitation system. (a) Conventional precipitator passage; (b) Idealized equivalent passage of (a); (c) Collector passage formed by opposite charged plates; (d) Passage for collection by image force.

at the wires, and $\quad @\ y^* = 1, \quad \dfrac{\partial \rho_p}{\partial y^*} = (1 - \sigma)\alpha_e \rho_p$

at the wall where deposition is desired. σ ranges from 0.1 to 0.5 at a clean surface in an electrostatic precipitator.

The influence of other fluxes at the boundary is neglected at present. A solution of [10] by separation of variables was given by Soo & Rodgers (1971). It is interesting to note that:

(a) At small drift force or small α_e and small σ, the density distribution is approximated by:

$$\rho_p/\rho_{p1} \simeq [1 + \alpha_e y^*] \exp[-\sigma\alpha_e Pe^{-1} x^*] \qquad [12]$$

for particle cloud density ρ_{p1} at the inlet. The collection efficiency η_c is obtained by integrating the total mass rate of particles:

$$\begin{aligned} \eta_c &\simeq 1 - \exp\left[-\sigma\alpha_e Pe^{-1}\frac{L_e}{b}\right] \\ &= 1 - \exp\left[-\sigma\left(\frac{q}{m}\right)\frac{E}{FU}\frac{L_e}{b}\right] \end{aligned} \qquad [13]$$

for passage length, L_e. This is the Deutsch equation (White 1963) modified by the introduction of an adhesion probability.

(b) At large α_e, we get

$$\rho_p \propto [2y^* - \sigma_e y^{*2}] \exp(\tfrac{1}{2}\alpha_e y^* - \tfrac{1}{4}\alpha_e^2 Pe^{-1} x^*) \qquad [14]$$

and
$$\eta_c \simeq 1 - \exp\left[-\tfrac{1}{4}\alpha_e^2 Pe^{-1}\frac{L_e}{b}\right]$$
$$= 1 - \exp\left[-\left(\frac{q}{m}\right)^2 \frac{E^2 b}{4F^2 D_p}\frac{L_e}{Ub}\right]. \quad [15]$$

Hence, for large α_e, the collection is not strongly influenced by the adhesion probability; the influence of particle diffusivity is readily seen. Space charge effect is neglected in the above. which is appropriate for a dilute suspension.

(c) Collection in the passage such as is shown in figure 1(d) depends on space charge effect when there is substantial concentration of charged particles. It depends on image force when the suspension is dilute. Because of the presence of particle diffusivity, even in laminar fluid flow in the passage, a particle is not expected to remain at the mid-plane indefinitely, and collection is produced by the interaction of diffusion and the image force (per unit mass) f_y which gives a drive velocity

$$\frac{f_y}{F} \simeq \frac{q^2}{\pi\varepsilon_o m F b^2}\frac{y^*}{(1 - y^{*2})^2} \quad [16]$$

where ε_o is the permittivity of free space. For a given adhesion probability σ, as a particle reaches $y = b - a$ (a is the particle radius):

$$\rho_p \simeq \rho_{pl}\exp\left[-\sigma\bar{\alpha}Pe^{-1}(a^2/b^2)x^*\right] \quad [17]$$

where
$$\bar{\alpha} = \frac{\bar{\rho}_p}{\varepsilon_o}\left(\frac{q}{m}\right)^2 \frac{b^2}{FD_p}$$

based on space charge effect of the material. The efficiency is given by:

$$\eta_c = 1 - \exp\left[-\sigma\frac{\bar{\rho}_p}{\varepsilon_o}\left(\frac{q}{m}\right)^2 \frac{a}{Fu_o b}\frac{a}{b}\frac{L_e}{b}\right] \quad [18]$$

where ρ_p is assumed to be uniform at each section of x because of high particle diffusivity. Equation [18] is a modified Deutsch equation for collection by image forces. The influence of other factors in the boundary condition [4] is illustrated in the following example:

Example: The following is given with a small plate-to-plate spacing: $\sigma = 0.5$, $\bar{\rho}_p = 10^3$ kg/m^3, $(q/m) = 10^{-1}$ coul/kg, $a = 1\ \mu$m, $F = 10^5$ sec^{-1}, $U = 3$ m/sec, $b = 4$ mm, $\eta_c = 95$ per cent. Case (a) at $V_o = 10^3$ v. requires the passage length L_e of 48 mm. Case (c) requires $L_e = 6.36$ m; however, for $a = 5\ \mu$m, $F = 4 \cdot 10^3$ sec^{-1}, case (c) requires $L_e = 10.2$ mm only.

6. CYCLONE SEPARATORS

The basic relations also make it possible to analyze more rigorously the performance of a cyclone separator. As shown in figure 2, a common configuration has dusty gas entering

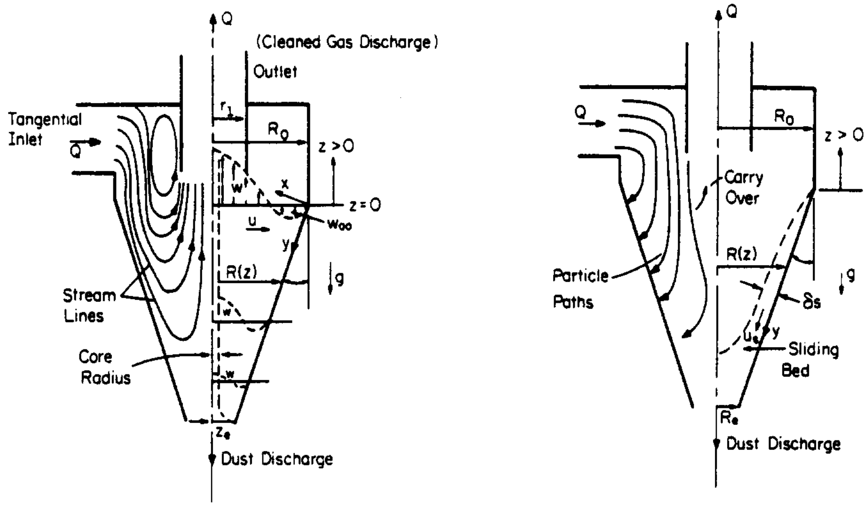

Figure 2. Conditions inside a cyclone separator and coordinates. (a) Fluid velocity distribution; (b) Particle path and sliding bed.

tangentially at the top of a cylindrical section to produce the vortex motion. The particles collected by the combined centrifugal and gravitational forces are removed at the bottom of the conical section. The cleaned gas exits from the top.

Figure 2 shows the coordinates and dimensions of the flow system for volumetric flow rate Q; radius $R(z)$ of the inside surface with coordinate r, z in the radial and axial directions respectively; fluid velocities u, v, w in the u_p, v_p, w_p for the particulate phase of a given species.

The density distribution of the particulate phase is strongly influenced by a finite particle diffusivity D_p and field forces, together with finite interaction length L_p and sticking probability σ of particles at the wall or with the deposited layer of particles (Soo & Tung 1972). The field forces include centrifugal, gravitational and electrostatic forces. The effect of electrostatic charges is prominent in gaseous suspensions. The electric charge effect is such that much of the carryover into the outlet pipe occurs due to electrostatic repulsion rather than by turbulence alone.

For the flow system shown in figure 2(a), the fluxes due to field forces in the diffusion equation are given by the equations of radial and axial components of particle momentum. In these equations the inertial forces may be neglected in comparison with field forces and viscous forces (Soo 1967). The electric field $\mathbf{E}$ is given by the Poisson equation.

At the boundary, for deposits of small thickness or for a clean wall, we have for coordinate x in figure 2(a) and gravitational acceleration g,

$$-D_p \frac{\partial \rho_p}{\partial x}\bigg|_R = (1-\sigma)\left[\frac{g}{F}\sin\phi + \frac{v_{pw}^2}{RF}\cos\phi\right]\rho_{pR} - [(1-\sigma)\rho_{pR} - \sigma'\rho_{pb}]\left(\frac{f_L}{F}\right) + (1-\sigma)\frac{q}{m}\left(-\frac{E_x}{F}\right)\rho_{pR} - \sigma_w\rho_{pr}\left(\frac{f_w}{F}\right). \qquad [19]$$

The diffusion equation can be expressed in dimensionless form by introducing $W = Q/\pi R_o^2$, where R_o is the largest radius of the cyclone:

$$r^* = r/R, \quad z^* = z/R, \quad u^* = u/W, \quad w^* = w/W, \quad v^* = vR/C, \quad V^* = V\bigg/\left[\frac{\rho_{pl}}{\varepsilon_o}\left(\frac{q}{m}\right)R_o^2\right],$$

$$\mathbf{E}^* = \mathbf{E}\bigg/\left[\frac{\rho_{pl}}{\varepsilon_o}\left(\frac{q}{m}\right)R_o\right], \qquad \rho_p^* = \rho_p/\rho_{pl} \qquad [20]$$

where C is the maximum vorticity of the system and V is the electric potential giving

$$\left(Pe\,u^* + \alpha E_r^* + \Omega\frac{v_p^{*2}}{r^*}\right)\frac{\partial\rho_p^*}{\partial r^*} + (Pe\,w^* + \alpha E_z^* - \gamma)\frac{\partial\rho_p^*}{\partial z^*} + 2\Omega\rho_p^*\frac{v_p^*}{r^*}\frac{\partial v_p^*}{\partial r^*} = \nabla^{*2}\rho_p^* - \alpha_p^{*2} \qquad [21]$$

where $$Pe = \frac{WR_o}{D_p}, \quad \alpha = \frac{\rho_{pl}}{\varepsilon_o}\left(\frac{q}{m}\right)^2\frac{R_o^2}{D_pF}, \quad \Omega = \frac{C^2}{R_o^2D_pF} \quad \text{and} \quad \gamma = \frac{gR_o}{D_pF}$$

correlating convection, diffusion, and transport by various forces in relation to diffusion and relaxation phenomena. The fact that the electrostatic, centrifugal and gravitational forces give rise to drift components on particles in addition to u and w is thus shown. Ω plays an important part at the wall because of large relative velocity v_{pw} given by [8].

When applied to a steep cone (small ϕ in figure 2), the local condition can be determined by integrating [21] with the simplification of small axial electrical field (compared with the radial electric field; i.e. $E_r^* \gg E_z^*$), nearly uniform axial velocity, or $w^* \simeq$ constant; small gravitational effect due to turbulence, $\gamma \simeq 0$; and small radial velocity (or $u^* \simeq 0$ because $u^* \ll w^*$). Integration from zero to r^* gives:

$$r^*\frac{\partial\rho_p^*}{\partial r^*} - \Omega\rho_p^*v_p^{*2} - \alpha\rho_p^*\int\rho_p^*r^*\,dr^* - \alpha_e\rho_p^* = Pe\frac{\partial}{\partial z^*}\int\rho_p^*w_p^*r^*\,dr^* \qquad [22]$$

where $$\alpha_2 = \frac{r_oE_e}{FD_p}\frac{q}{m}$$

E_e is the external field due to potential or surface charge density of the wall, and r_o is a characteristic dimension for the applied field depending on its geometry. The right-hand side of [22] is the dimensionless rate of deposition of particles.

With these simplifications, and taking into account the effects in [19], the collection efficiency η_c over a height L_c of the cyclone is given by integrating [22]:

$$\eta_c \simeq 1 - \exp\left\{-2\sigma[\Omega + 4(2 - k^*)\alpha + \alpha_e]\frac{L_c}{R}Pe^{-1} + 2\left[\sigma + \sigma_w'\frac{\rho_{pb}}{\rho_{pw}}\right]\frac{f_L}{F}\frac{L_c}{D_p}Pe^{-1} - 2\sigma_w\frac{f_w}{F}\frac{L_c}{D_p}Pe^{-1}\right\}. \qquad [23]$$

Using the approximation for the density distribution due to centrifugal force alone:

$$\rho_p^* \simeq \rho_{pR}^* \exp\left[-\frac{\Omega}{2}\left(\frac{1}{r^{*2}} - 1\right)\right] \quad [24]$$

and

$$k^* = (\Omega/2)\exp(\Omega/2)\int_{\infty}^{\Omega/2} x^{-1}e^{-x}\,dx \leqslant 0(1)$$

for $\Omega > 0(10)$ and $\rho_{pR}^* = \rho_{pR}/\rho_{pl}$. Equation [23] shows the influence of f_L in decreasing the efficiency. Neglecting f_L and f_w, [23] can also be expressed for large Ω as:

$$\eta_c \simeq 1 - \exp\left\{-\sigma\left[\left(\frac{2\pi C^2 L_c}{FQR_o^2}\right) + 8\pi\frac{\rho_{pl}}{\varepsilon_o}\left(\frac{q}{m}\right)^2\frac{R_o^2}{F}\frac{L_c}{Q} + 2\pi E\left(\frac{q}{m}\right)\frac{R_o L_c}{FQ}\right]\right\} \quad [25]$$

for length L_c and volume flow rate Q. Note that $C^2L_c/18FQR_o^2$ is just the empirical cyclone number of Rietema & Verver (1961) and $9C/FR_o^2$ is the empirical Tengbergen group (Leith & Licht 1972). Some agreement with experimental results using these dimensionless groups is hence not surprising. Other terms in [25] show that by a combination of centrifugal force and electrostatic forces, a large diameter ($2R_o$) cyclone separator may have the same efficiency as a small one, for similar pressure drop of the gaseous suspension. This is not the case for hydrocyclones.

Example: A system with $2R_o = 0.25$ m, $Q = 0.187$ m²/sec, $C = 4.17$ m²/sec, $L_c = 0.0875$ m, $2a = 10\,\mu$m, $\bar{\mu} = 4\cdot10^{-6}$ kg/m/sec, $\bar{\rho} = 5$ kg/m³, $F = 500$ sec^{-1}, $\rho_{pl} = 2.98$ kg/m³, $q/m = 10^{-4}$ we get $\eta_c = 97.3$ per cent; for $2R_o = 1.5$ m, $Q = 6.43$ m³/sec, $L_c = 5.25$ m, and similar pressure drop, the dimensionless groups are 0.32 and 77.8, respectively, giving $\eta_c = 98$ per cent.

In the absence of surface adhesion, sedimentation occurs when ρ_{pb} is reached at the wall. For volume fraction solid, $\Phi_s = \rho_{pb}/\bar{\rho}_p$ for sedimentation, and $\Phi_1 = \rho_{pl}/\bar{\rho}_p$ at the inlet, sedimentation begins at $(\alpha/8)\Phi_1 > 1 - \sqrt{(\Phi_1/\Phi_s)}$, or, for $\rho_{pl}/\bar{\rho}_p \ll 1$,

$$\bar{\alpha} = \frac{\bar{\rho}_p}{\varepsilon_o}\left(\frac{q}{m}\right)^2\frac{R_o^2}{D_pF} > \frac{8}{\Phi_1} \quad [26]$$

where $\bar{\alpha}$ is the value of α where ρ_{pl} is replaced by $\bar{\rho}_p$. $\bar{\alpha}$ gives the minimum q/m for sedimentation by electrical effect. The most desirable condition for collection in a cyclone separator is where the adhesion probability is zero but a bed of density ρ_{pb} flows down the cone.

The ideal situation would be a dense bed (collection by either the centrifugal or the electric field) flowing down the cone toward the dust discharge, figure 2(b).

Unloading of the collected particles at the bottom of the cyclone is similar to the condition of unloading of bins and flow of solids through an orifice; namely, the pressure is often lower in the inside of the bottom cone. The difficult problem is usually to get the collected lay to slide down from the upper portion of the cone.

For a sliding bed of collected particles, the integration of the equation of bed momentum for a steep cone results in:

$$\left(\frac{\delta_s}{R_o}\right)^3 \simeq \left(\frac{\sigma\mu_s D_p}{g\rho_{ps}R_o^3}\right)\left(\frac{\rho_{pl}}{\rho_{ps}}\right)\left(\frac{y}{R_o}\right)\frac{(\Omega + \pi\alpha)}{(\cos\Phi - f\sin\phi)} \quad [27]$$

or $\delta_s \propto (-z)^{1/3}$, (negative z points downward, where $\delta_s = 0$ at $z = 0$), ρ_{ps} is the sliding bed density, f is the Coulomb friction coefficient at the wall, μ_s is the shear resistance of the sliding bed, ρ_{pl} is the inlet density of the particles, and $\bar{\rho}_p$ is the density of solid material. δ_s increases toward the bottom of the cone. For a thin layer of deposit, at a steady rate of removal, the weight is balanced by a shear stress,

$$\tau_s = \delta_s \rho_{ps} g (\cos \phi - f \sin \phi). \qquad [28]$$

To unload, τ_s must be greater than the yield stress of the bed. Larger yield stress leads to a thick bed. The shear stress due to fluid flow is, in general, negligible.

For the example of a cyclone of $2R_o = 1.50$ m, $\rho_{ps} = 822$ kg/m^3 (0·6 fraction solid) and a yield stress of 476 n/m^2, [28] gives $\delta_s \simeq 0.0672$ m, a thick layer. Successful operation calls for small yield stress, which depends on consolidating pressure produced by the centrifugal and electrostatic forces. Large consolidating pressure and high moisture content cause large yield stress.

The worst situation is when the electrical resistivity of particles is high such that a tenacious insulating layer is formed with a large surface charge density. In this case, previously collected particles do not unload, and carry-over to the outlet occurs.

7. DISCUSSION

The above formulations show how lift forces, bed erosion, and splashing act on the collection efficiency. The influence of the surface force is such that collection occurs by diffusion alone in the absence of field forces. Although this influence is usually small in comparison with field forces, it is not as easily controlled.

The electrical effect in a cyclone separator for a gas–particle suspension is usually significant; the possibility of applying an electric field to enhance collection is indicated.

For all collection devices, a knowledge of adhesion probability of particles to the surface of the system is important. More data are needed.

Formulation of the phenomena made possible identification of a minimum number of dimensionless groups for correlating data and for scaling. These are: Kp, S, Pe, α, $\bar{\alpha}$, α_e, Ω, γ.

The differential equations also make possible numerical calculations for a detailed design and numerical simulation of these devices.

For a suspension with a particle size distribution function $f(a_s)$, the overall efficiency is given by:

$$1 - \eta_c = \int_{a_1}^{a_2} (1 - \eta_{cs}) f(a_s)\, \mathrm{d}a_s \Big/ \int_{a_1}^{a_2} f(a_s)\, \mathrm{d}a_s$$

for ranges of particle sizes from a_1 and a_2. Splashing of a deposited layer by species of large L_p should be accounted for even for a dilute suspension.

The problem of removal of particles in lumps from the collecting surface is at least as important as the collection of individual particles. The treatment of sliding bed for the case of the cyclone separator can be extended to the case of rapped collector plates in a conventional electrostatic precipitator. The entrainment by such a falling cloud of dust should also be accounted for in the boundary condition [4].

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Sommaire—On analyse l'enlèvement de particules d'une suspension de gaz par une surface, et la régénération subséquente de la surface. Le dépôt des particules dans le dispositif de collection est dû principalement au champ électrique (dans les précipitateurs électrostatiques) et à une combinaison de champs électriques et centrifuges (dans un séparateur cyclonique). La bonne conception de dispositifs de séparation de particules, y compris la régénération de surface, est alliée à ces forces de champs et à d'autres effets.

Auszug—Es wird die Entfernung von Partikeln aus einer Gassuspension via einer Oberfläche und die nachfolgende Regeneration der Oberfläche analysiert. Die Abscheidung der Partikel in der Auffangvorrichtung erfolgt vorwiegend durch das elektrische Feld, (in elektrostatischen Ausfällapparaten) und durch eine Kombination von elektrischen und Zentrifugalfeldern, (in einem Zyklonabscheider). Korrekter Entwurf dieser Abscheidungsvorrichtungen von Partikeln, einschließlich Oberflächenregeneration, wird auf diese Feldkräfte und auf andere Wirkungen bezogen.

Резюме—Анализируется удаление взвешенных частиц из газа через поверхность и последуюшее восстановление поверхности. Отложение частиц в сборном приспособлении в первую очередь происходит вследствие электрического поля (в электростатических отстойниках) и благодаря комбинации электрического и центробежного полей (в циклонном сепараторе). Проектирование подходяших приспособлений для отделения частиц, включая восстановление поверхности, зависит от этих сил поля и от других эффектов.